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Particle Filtering of Radar Signals for Non-Cooperating Target Imaging

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Abstract. This paper describes the application of the optimal nonlinear/non-Gaussian filtering theory to the radar signal processing problem. This approach, made feasible by a new technique named *Particle Filtering*, may cope with nonlinear models as well as non-Gaussian dynamic and observation noises. The *Particle Filter* constructs the conditional probability of the state variables, with respect to the measurements, through a random exploration of the state space by particles, which obey the conditional probability generator. The application of this new filter to the inverse synthetic aperture radar (ISAR) technique allows the joint estimation of the path and the image of a maneuvering target in weak signal to noise ratio situations.

Keywords: Inverse Synthetic Aperture Radar, ISAR, Radar Imaging, Nonlinear Filtering, Particle Filtering.

1. Introduction

Nowadays, the synthetic aperture radar is the main technique adopted to image moving targets, like airplanes, with a ground radar. This technique, that exploits the relative motion between target and radar, is based on the coherent sum of a sequence of backscattered radar pulses, each pulse corresponding to a different angular position (attitude) of the target.

Usually, the image reconstruction of a fixed target by a moving radar is named synthetic aperture radar (SAR) while the symmetric case, i.e., imaging a moving target with a fixed radar, is called *inverse* SAR (ISAR). In both situations, the knowledge of the relative radar-target motion is essential to the imaging algorithm. In SAR problems, one normally knows the (nominal) trajectory of the on board radar and can use external information (such as an inertial navigation system) or some autofocusing technique to motion compensate the image data (Buckreuss,1991; Moreira, 1989). Imaging a moving, non-cooperating target, where the path is *a priori* unknown or poorly determined, is a bit more involved. Specific trajectory estimation algorithms need to be developed to track maneuvering targets and compensate the radar signals. Once the trajectory is estimated, one can phase compensate the returned pulses and coherently sum them to exploit the imaging capability of synthetic aperture techniques.

Most of the actual ISAR algorithms utilizes this approach, i.e., some kind of data preprocessing is done to ensure separability between tracking and imaging problems. The main difficulty with this approach is the need of a stable strong scatterer on the target that can be tracked as a reference point. In a complex target, such a reference point can disappear during the observation time or jump abruptly from one strong point to another.

The new method proposed in this paper, based on nonlinear filtering theory, allows simultaneous radar tracking and imaging of a complex non-cooperating target. This technique can integrate the extended nature of the target in the tracking algorithm and needs no isolated strong scatterer as a reference.

The organization of the paper is as follow. In section 2, we develop the models for target movement, target electromagnetic response and radar measurements which are needed to state the radar signal processing as a filtering problem. Particle Filtering is introduced in section 3. Tracking and imaging are then considered in Section 4 using the models and the algorithm described in sections 2 and 3. In section 5 we present some simulation results and the conclusion is drawn in section 6.

2. Modeling

2.1- The Spatial Target Model

Most imaging algorithms use (sometimes implicitly) the so-called "weak scatterer" approximation to represent the target to be imaged (Chassay, 1983; Borden, 1994). In this approximation, the target is equivalent to a set of point-like scatterers and the backscattered electrical field $\mathbf{E}_{scat}$ is the superposition of each individual scatterer response to the incident electrical field. Interaction among scatterers, multipath, shadow effects and diffraction are disregarded. Moreover, the backscatter coefficients σ_i are assumed to be isotropic (invariant for small changes of target attitude during the considered portion of flight). That is the simplest geometrical optics model for the backscattering phenomenon.

With these assumptions, the target can be modeled by a grid of N elementary scatterers with backscatter coefficients σ_i:

$$T(\vec{x}) = \sum_{i=1}^{N} s_i \, d(\vec{x} - \vec{x}_i) \tag{2.1}$$

where $\delta(\cdot)$ is the Dirac measure, σ_i is a complex constant and $\mathbf{x}_i$ is the position of the scatterer. Complex coefficients σ_i indicate local amplitude and phase of the backscatter field.

More sophisticated models can be envisaged to take into account scattering perturbation effects, that are frequency- (f) and target attitude- (θ) dependent. In this paper we shall consider a narrow band signal, and dependency with respect to θ along the portion of target movement may appear as a random drift with suitable characteristics. Despite these simplifications, the model accounts for important phenomena found in radar signal processing, such as glint, and constitutes a basic approximation for the radar imaging problem.

2.2- Coherent Radar Signal

Consider that the transmitted signal is given by

$$s(t) = h(t)\Re e\left\{\exp(jw_0 t + j_0)\right\} \tag{2.2}$$

where $f_0 = \omega_0/2\pi$ is the carrier frequency, h(t) is the signal envelope and φ_0 is the phase of the transmitted signal. If the target is modeled by the equation 2.1, the amplitude of the scattered field can be written as

$$Y(t) = \sum_i \frac{K}{R_i^2} G(\theta_i) h(t - \tau_i) \sigma_i \exp\left[j\omega_0 (t - \tau_i) + j\varphi_0 \right] \tag{2.3}$$

with $\tau_i = 2R_i/c$ the signal delay for the scatterer located at a distance R_i to the radar, $G(\theta_i)$ the antenna gain in the direction θ_i and $1/R_i^2$ the free space attenuation of the signal amplitude. After carrier suppression by complex demodulation one gets:

$$Y(t) = \sum_i \frac{K_i}{R_i^2} h(t - \frac{2R_i}{c}) \sigma(R_i, \theta_i) \exp\left[-j\frac{4\pi}{\lambda} R_i \right], \quad K_i = KG(\theta_i)\exp(j\varphi_0) \qquad (2.4).$$

For a rigid target, we can consider an arbitrary reference point (R_0, θ) and describe the scatterers distribution in a target coordinate system (**figure 2.1**). Therefore, a target point i= (x,y) can be represented by

$$R_i = R(x,y) = R_0 + x \cdot sin\theta + y \cdot \cos\theta$$

under the assumption that target span is much smaller than R_0. That is the plane wave far-field approximation. For sake of simplicity we don't indicate the time dependency of R_0 and θ.

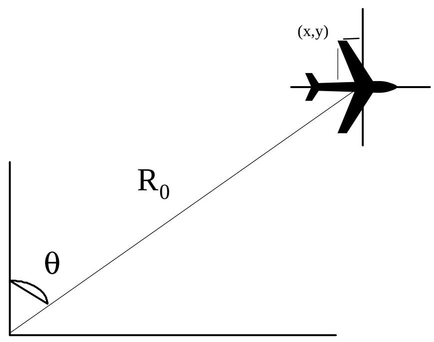


Fig. 2.1- Relation between radar and target coordinate systems.

Now, equation 2.4 can be rewritten as

$$Y(t) = \sum \frac{K_i}{R_i^2} h(t - \frac{2\left[R_0 + x.sin\theta + y.\cos\theta\right]}{c}) \sigma(x,y) \exp(-j\frac{4\pi}{\lambda}\left[R_0 + x.sin\theta + y.\cos\theta\right]) \tag{2.5}$$

Moreover, a noise *v*(t) adds to this returned signal. With the usual assumptions on the matched input filter and optimal sampling, this noise can be modeled as a complex white Gaussian process with zero mean and variance E(*vv**)= R, where *v** indicates the transpose complex conjugate of *v*.

Note that distance variations among different scatterers i= (x,y) are mainly important at signal phase level, with little impact on signal amplitude. We can therefore simplify the equation above by neglecting amplitude fluctuation due to the term K_i/R_i^2 and approximate it by K_0/R_0^2, where

$K_0 = G(\theta_0).\exp(j\varphi_0)$. At this stage K_0 appears as a single phase/amplitude reference, which may be discarded by immersion into $\sigma(x,y)$ when necessary.

Equation 2.5 allows us to retrieve several classical radar imaging formulations, as tomographic reconstruction and Fourier transform techniques (Mensa,1991). All these techniques suppose an *a priori* knowledge of target path, i.e., of R_0 and θ as a function of time. For non-cooperating targets, trajectory must be accurately estimated (with relative variations not exceeding a fraction of wavelength) before image formation. Next section presents some popular methods for separate target motion estimation, before we introduce the joint tracking/imaging procedure.

2.3- Motion Compensation

Synthetic aperture formation depends on a coherent sum of successive pulses backscattered by an uniform rotating target, placed at a constant distance of radar. Therefore, some kind of motion compensation is necessary if the target trajectory departs from this simple scheme. Motion compensation is normally divided into two steps: range bin alignment and phase compensation.

Range bin misalignment is due to radial motion of the target. Scatterers travel several range bins during the observation time, so that signals in a specific bin correspond to different scatterers. A correction step is needed to keep the scatterers in their initial range bins. To cope with the range misalignment problem, most algorithms use the correlation between adjacent returned pulses, as is done by the Spatial Domain Realignment and Frequency Domain Realignment algorithms proposed in (Chen and Andrews,1980) or by the synthetic reference envelope algorithm proposed in (Delisle and Wu,1994).

Concerning transversal motion, the target induces two kinds of phase variation: motion of the target center along radar line-of-sight, and rotation relative to the target center as viewed from the radar. Only the target rotation creates cross-range resolution (differential Doppler) and a phase compensation algorithm is required to eliminate the translational motion effect.

Most compensation techniques propose to track a strong, steady scatterer on the target that can be used as a reference point for the target path. This reference point may correspond to an isolated scatterer, like a wing tip, that represents a peak in the signal return. One can also track the range bin where the normalized variance of the returned signal amplitude is minimal. This range bin is supposed to contain a strong scatterer, called dominant scatterer (Steinberg,1988), that gives a reference for the phase compensation.

2.3.1- Motion Model

None of the above techniques can be applied at low signal-to-noise ratio (track before detect), or with no dominant scatterer. We propose here a global nonlinear filtering approach to process the radar signals for simultaneous detection, tracking and imaging of complex targets.

To apply stochastic filtering techniques we need to model the motion as a dynamic random process and its measurements by the radar. This approach, which is embedded in usual radar tracking in a linear filtering stage, is considerably more sophisticated in nonlinear tracking, such as proposed in this paper.

Linear algorithms are used for smoothing models of target motion, with Gaussian assumptions for the driving process. The Singer model (Singer, 70) is the simplest of them for the tracking of maneuvering targets. It is a modified triple integrator system where the motion are considered independent in each Cartesian axis and driven by a correlated Gaussian acceleration. We use here (**figure 2.2**) a more realistic version of this model, where acceleration and speed are physically limited and maneuvers decisions are represented by a random point process.

Fig. 2.2 - Modified Singer model.

The system is driven by the doubly stochastic process π_t that adds random jumps to the acceleration to represent realistic maneuver controls. These maneuvers follow a Poisson process with mean time T_m between jumps. Their amplitude is normally distributed with variance σ_Q^2. Actual control is limited by saturation to take into account physical constraints on target thrust and maneuvering capabilities.

3. The Particle Filter

Nonlinear filtering is the natural frame to state global estimation problems where a dynamic stochastic process (here the target motion) is partially observed (by a nonlinear measuring device - the radar) and corrupted by an additive stochastic process (the noise). The difficulty with nonlinear filtering formulation comes from the infinite dimensional character of the solution, which can not be derived in closed form. Approximate solutions based on local linearisation of system and measuring equations and/or moment truncation of density probability can not maintain guaranteed performance or even stability of the filter.

The particle filtering technique (Rigal, 1993; Noyer, 1996) may cope with nonlinear models as well as non-Gaussian dynamic and observation noises. It recursively constructs the conditional probability measure $dP(x_t \mid y_0^t)$ of the state variables x_t, with respect to all available measurements y_0^t, through a random exploration of the state space by entities called particles. Particles obey the conditional probability generator which involves a Bayes correction term based on measurements. Its main advantage relies on probabilistic properties of the procedure, which lead to global convergence.

Particle filtering works in a evolution/correction basis dictated by the system equations. Each particle simulates an admissible trajectory (candidate) of the state variables followed by a correction step due to measurements. As the number of particles increases, the particle filter converges to the optimal state estimator. Uniform convergence has been shown for one of the versions of the algorithm in (Del Moral and Salut,1995).

3.1- Particle Filter Algorithm

Let the following equations represent a discrete dynamic system X with Y as a measurement:

$$\begin{cases} X_{k+1} = f(X_k, k, \pi_{k+1}), & X_0 \textit{ initial condition} \\ Y_k = h(X_k, k) + \nu_k \end{cases} \tag{3.1}$$

where π_k and v_k are independent white noises with known probability distributions.

Particle filtering approximates the probability $dP(x_0)$ at initial time by a set of *N* Dirac distributions and applies the dynamic of the system to this set. In other words, one "randomly selects N particles" from the probability distribution $dP(x_0)$ to represent it as:

$$dP(x_0) \approx \frac{1}{N} \sum_{i=1}^{N} \delta(x_0 - x_0^i) \tag{3.2}$$

where δ is the Dirac delta.

Next, each particle i follows the system dynamic $f(x_0^i, 0, \pi_0^i)$ with noise samples π_0^i generated from its a priori probability. Time evolution of Dirac point measures is given by sequential application of this procedure. **Figure 3.1** shows the evolution of a set of particles. Dot lines represent the trajectory of each particle.

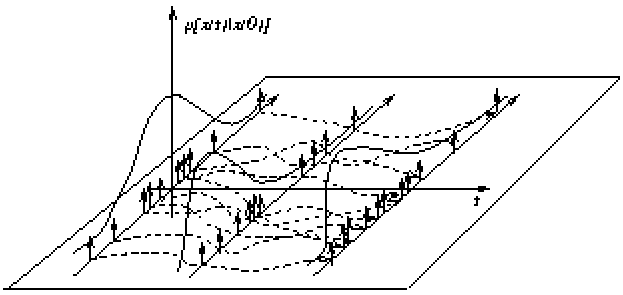


Fig. 3.1 - *A priori* exploration.

Next step is to introduce information carried by the measure y_0^k. This is done by the term $p(y|x_i)$ in the Bayes's theorem. Information given by measure y_k "weights" the trajectory of each particle i. The particle estimation (with *N* particles) of any measurable function $\varphi(\cdot)$ of the state X_k is given by:

$$\hat{\varphi}^N(X_k) = \sum_{i=1}^{N} p_k^i \varphi(x_k^i), \qquad with\ p_k^i = \frac{Z_k^i}{\sum_{j=1}^{N} Z_k^j} \quad and\ Z_k^i = \prod_{t=1}^{k} p(y_t | x_t^i) \tag{3.3}$$

Therefore the "weight" p_k^i corrects the representation given in **figure 3.1**, where all particles had the same weight 1/N. The particle estimation of the conditional probability is showed in the **figure 3.2**. Here, dot lines give the trajectories and the arrows' amplitude represents the weight of each particle.

FIG. 3.2

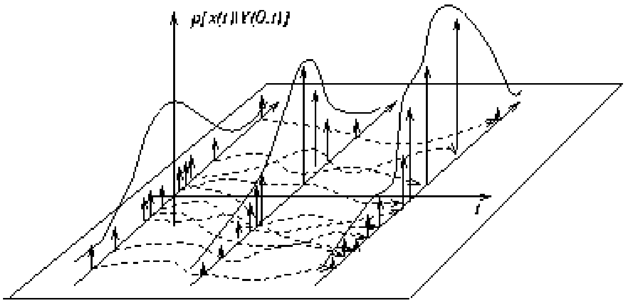


Fig. 3.2 - Conditional probability.

3.2- Regularisation Techniques

If the state space is unbounded (as is the case for target positions), particle trajectories diverge. Furthermore, in the absence of regularisation, particle weights degenerate and the law of large numbers is no more applicable. This second phenomenon is due to the finite number of particles N used in the algorithm. To cope with this, some kind of regularisation must be applied. A possible technique, in the first case, is the forgetting of old data, frequently used to adapt filter parameters to unknown system's evolution or poor modeling.

We use here a resampling technique that solves both difficulties at the same time. The algorithm is restarted at an instant t using the estimated conditional probability as an initial distribution. All particles are redistributed among the states x_t^i according to the weight attributed to them. All particles take the same weight 1/N after the redistribution.

Therefore, most probable states, corresponding to "heavy" particles, give rise to several new particles while least probable ones are "killed". Redistribution locates particles where they are needed, in a probabilistic way.

4. Tracking / Imaging Particle Algorithm

As indicated in section 2.3, synthetic aperture imaging can be applied if the target trajectory is a priori known. In this case one can compensate the translation motion and just keep the rotational motion about a reference point, that induces differential Doppler, to obtain cross-range resolution.

Classical imaging techniques usually estimate target trajectory *before* imaging. As target motion/image estimation from radar measurements is a nonlinear operation, such a separation approach is not optimal, as can be noticed in practice when glint is present in radar data.

Particle filtering can be applied to jointly estimate motion and image, using optimally the available information. Glint and other interference effects are eliminated or greatly reduced by modeling and processing a multi-scatterer target that take into account the extended nature of the target.

4.1- Image Formation

As one can see in equation (2.5), measurements Y(t) are linearly related to backscatter coefficients σ(x,y), for a given target trajectory. Sampling of Y(t) results into a data vector where each sample corresponds to a range bin, with radial resolution c/2B (c - light speed; B - receiver bandwidth).

Time discretisation gives a nonlinear system of equations

$$Y(t) = H_t(x,y)D_t(e^{-jf})s \tag{4.1}$$

where each line of matrix H corresponds to the amplitude of the scatterers in a range bin and D is a diagonal matrix with phase terms exp[-jΦ(x,y)], Φ(x,y)= (4π/λ)R(x,y).

As previously mentioned, this structure of radar signal allows to compute σ(x,y) as a linear estimation conditionally to the trajectory R_t. Consequently, each particle i in the algorithm is associated to:

- an estimated trajectory, according to the motion model of the target;
- a grid of points (x,y), moving with the particle and representing the target's reflectivity model;
- a linear estimator of the target image $\sigma_i(x,y)$ for the grid points;
- a probabilistic "weight", given by the Bayes correction.

4.1.1- Conditional Linear Filter

Consider a grid of points (x,y) whose center follows a given trajectory R_t. Along this trajectory, the radar imaging reconstruction can be viewed as the solution of the following filtering problem (Chamon, 1996):

$$\begin{cases} s_{t+1} = s_t & s_0 \approx N(0, P_0) \\ Y_t = H_t(x,y)D_t(e^{-jf})s_t(x,y) + v_t \end{cases} \tag{4.2}$$

The image $\hat{s}(x,y)$ is given by the following regularized pseudo-inverse:

$$\begin{cases} \hat{s}_{t+1} = P_{t+1}\left(\sum_{t=0}^{t+1} D_t^* H_t^* R^{-1} Y_t\right) \\ P_{t+1}^{-1} = P_0^{-1} + \sum_{t=0}^{t+1} D_t^* H_t^* R^{-1} H_t D_t \end{cases} \tag{4.3}$$

where A* is the transpose conjugate of A and R= E(vv*) is the noise covariance matrix. For a uniform array of N antennas this solution generalizes in a straightforward way to

$$\hat{s}_t = \left(\sum_t D_0^* H_0^* H_0 D_0 + \frac{rP_0^{-1}}{N}\right)^{-1} \sum_t D_0^* H_0^* \frac{\sum_{n=0}^{N-1} Y_n e^{-jnf}}{N} \tag{4.4}$$

Here time dependence τ is not indicated, H_0D_0 represents the first antenna in the array and R= rI is the noise covariance, with I the identity matrix. The term ϕ_n, that represents the phase delay to the n-th antenna, can be written in terms of the phase of first antenna as $\phi_n = \phi_0 - (2\pi d/\lambda).n.\sin\theta$, where *d* is the inter-element spacing in the array and θ is the direction of signal arrival.

This result shows that σ estimation is obtained by coherent summation of data both in space (sum over N antennas) and in time (time variation of H, D and ϕ). Phase equalization of data over the array (the term $Y_n e^{-j\phi n}$) just points the antenna diagram to the target.

4.2- Motion Estimation

Motion estimation is accomplished by the particle filter that uses the image reconstructed by the conditional filter to calculate the likelihood (or equivalently the "weight") of each particle trajectory in state space. We can note that target image and trajectory are jointly estimated by the filter: a "good" image indicates a "good" trajectory and conversely, a "good" trajectory yields an accurate image. The particle algorithm is schematically described in **figure 4.1**.

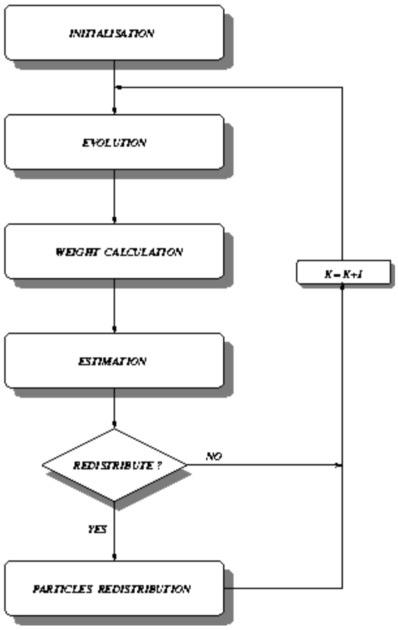


Fig. 4.1 - The particle algorithm.

5. Simulation Results

We presents in this section simulation results for the extended target indicated in **figure 5.1**. It represents the signal reflected by 10000 elementary scatterers with unity amplitude and random distributed phases.

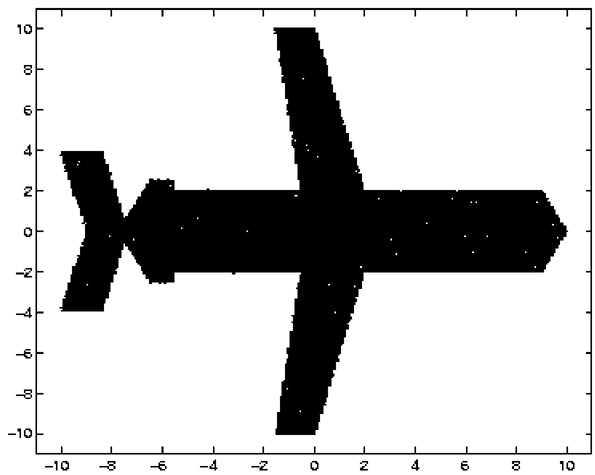


Figure 5.1- Target model used in simulations

The following parameters was used in the simulation:

a) Radar characteristics

Frequency	10 GHz
Pulse Duration	6.7 ns (1m range resolution)
Pulse Repetition Interval	0.2 ms
Antenna Array	Two omnidirectional antennas with spacing d= 50λ/2 (20 mrad beamwidth)
Signal to Noise Ratio	0 dB
Integration Time	3 s

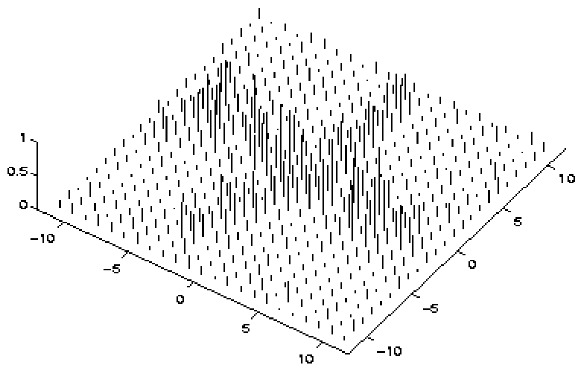
b) Trajectory characteristics

Parameter	Nominal Value	Initial Uncertainty
angular position	0 rad	+/- 1 mrad
distance	10000 m	+/- 5 m
velocity (angular direction)	250 m/s	+/- 20 m/s
velocity (radial direction)	-20 m/s	+/- 5m/s

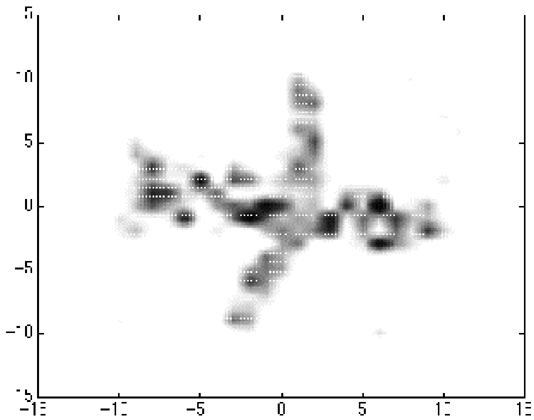
5.1- Radar Imaging with Known Trajectory

For a perfectly known trajectory, we show in the **figure 5.2** the result of the radar signal processed by the conditional filter. In the **figure 5.2a** we see the output amplitude for each point of the grid (x,y) that represents the target (note that the number of points in the grid is much smaller than that of the simulated target). **Figure 5.2b** shows the same data interpolated for a finer grid.

FIG. 5.2b



(a)



(b)

Figure 5.2- Image results with a known trajectory. a) Filter output. b) Interpolated image.

5.2- Radar Imaging of a Non-Cooperating Target

When the trajectory and the target backscatter coefficients σ_i are jointly estimated by the particle filter, we get the image indicated in **figure 5.3** below. The result, if compared with the images in **figure 5.2**, is necessarily less precise but we note that the regions of strong reflectivity are suitably estimated and the target's general shape is reconstructed.

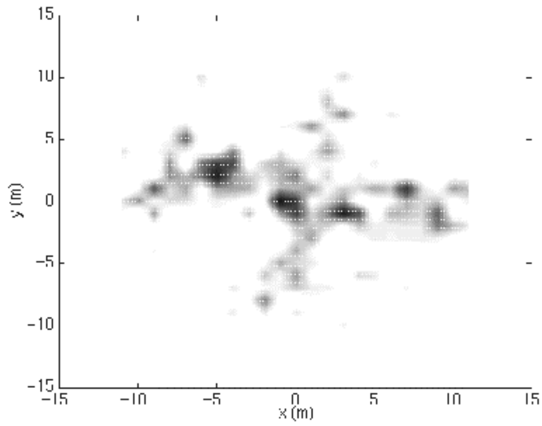


Figure 5.3- Image result with an estimated trajectory.

6. Conclusion

We have developed a new approach to filter radar signals and jointly estimate the path and the image of a maneuvering target. The proposed filter allows the processing of nonlinear/non-Gaussian models for target dynamics and represents an asymptotic approximation of the optimal solution for the general nonlinear filtering problem. Images obtained with this technique faithfully reconstruct the path and the general shape of the target. It's worthwhile to note that the same algorithm can be applied for trajectory estimation only of a complex target, filtering out glint and other scintillation phenomena. In this case, a coarse grid, representing just some strong scatterers, can be used in the target model, as detailed image is no longer required.

Because particle filtering makes use of a random exploration of the state space, computational cost is a main concern. In fact, each particle needs to calculate the pseudo-inverse of a huge matrix (associated to the grid that represents the target) to evaluate the "weight" of its trajectory. As particles evolve in a independent way, communicating only for redistribution and likelihood normalization, a parallel version of the algorithm is a current subject of interest to speed up the image estimation. Moreover, we may consider adaptive features, where the grid is refined as the trajectory is more precisely estimated.

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