An Adaptive Clustering MAP Algorithm to Filter Speckle in Multilook SAR Images

FÁTIMA N. S. MEDEIROS[1,3]
NELSON D. A. MASCARENHAS[2]
LUCIANO DA F. COSTA[1]

[1] Cybernetic Vision Group IFSC -University of São Paulo
Caixa Postal, 369, SP,13560-970,Brasil
e-mails: {luciano,fsombra}@ultra3000.ifsc.sc.usp.br

[2] UFSCar - DC
Via Washington Luiz Km 235, Caixa Postal 676
13565-905 - São Carlos, SP, Brasil
nelson@dc.ufscar.br

[3]UFCe- DEE
Campus do PICI Bloco 705
60455-760 - Fortaleza,CE, Brasil
fsombra@dee.ufc.br

Abstract. SAR images are generally affected by a multiplicative noise, called speckle, which degrades the quality of these images. Using this model we present an algorithm based on the Maximum a Posteriori (MAP) criterion to reduce speckle noise of multilook amplitude data. The speckle in these images is approximately described by the Square Root of Gamma distribution, which is used to develop MAP filters using different a priori distributions. We also suggest the combined use of MAP and the k-means clustering algorithm as a formal way to choose the best window size to update noisy pixels. We conclude this work by calculating the coefficient of variation, defined as the ratio of the standard deviation to mean, of MAP filtered images and of the original image to measure the reduction of the speckle in homogeneous areas.

Keywords: Multilook image, MAP estimator, k-means classifier, speckle noise, filtering.

I. Introduction

Speckle noise is one of the main characteristics present in images obtained by coherent imagery systems such as synthetic aperture radar (SAR), lasers, and ultrasound images. This kind of signal-dependent noise limits the visual interpretation of these images because it obscures the scene content. It is recommended to overcome this difficulty prior to classification procedures, for example. In such cases, filtering algorithms used as "a prior" step would improve the classification performance. In the literature, classical filters, like Lee (Lee, 1980), Kuan, (Kuan et al. 1985), Sigma (Lee, 1983), Frost (Frost et al., 1982), Median (Castleman, 1996) amongst others aim to reduce the noise speckle. The ideal filter must smooth the noise without eliminating radiometric and textural information that are fundamental for detail preservation (Lopes et al., 1988).

It has been experimentally verified in several works that for SAR images over homogeneous areas, the standard deviation of the signal is proportional to its mean Lee (1981). This fact suggests the use of the multiplicative model for the speckle and it was used by Kuan et al. (1987) to propose an adaptive non-linear pointwise filter that satisfies the MAP criterion for single look, quadratic detection and Gaussian “a priori” density.

The variance ratio of the original and noisy image is used as a measure of local properties by the adaptive filters to control filter window size (Li, 1988). By combining the MAP filter and the k-means clustering algorithm over Changle Li’s variance ratio (Li, 1988) it is possible to classify the noisy image in regions of homogenous statistics. In order to adapt the MAP filtering to the local statistics, the thresholds on the variance ratio are chosen to determine the window sizes for parameter estimation.

In practical applications the noise is often reduced by multilook processing, which can be done by averaging independent samples of several images. With an increasing number of averaged samples, the Rayleigh distribution of a signal approximates a Gaussian distribution (Hagg et al., 1996). Although improving the signal to noise ratio by $\sqrt{N}$, where N is the number of looks used to generate the image, this technique also diminishes the spatial resolution.

In this work the speckle distribution over multilook amplitude data is modelled by a Square Root of Gamma and we use it in the proposed MAP filter. The statistical parameters in the filtering algorithm are calculated by using a fixed window size (5x5) around each pixel or choosing a window size according to the degree of roughness of the non-noisy signal around the pixel. The clustering of the coefficients of variation determines the suitable filtering window. It will be shown in this article that this fact leads to a better filtering result.

In section II we present the multiplicative model for the speckle and derived from this model in section III the MAP estimator is formulated using the “a priori” Gaussian, Gamma, Chi-square, Exponential and Rayleigh densities. There is a brief discussion about the implementation results in section IV. Section V summarizes the conclusions and section VI outlines possibilities for future work.

II. Multiplicative Model and Speckle Statistics

The model used to describe the speckle is given in terms of a multiplicative noise given by equation (1), where z_A describes the amplitude SAR noisy image, x is the original signal and n_A is the noise with unitary mean and standard deviation σ_n. The multiplicative model is a good model over homogeneous areas because the standard deviation is proportional to the mean. The speckle noise and the original image are assumed to be decorrelated.

$$z_A = x.n_A \qquad (1)$$

Equation 2 represents the β index which is the ratio of the standard deviation to the mean used to measure the strength of the speckle in this kind of image and N is the number of looks.

$$\frac{s}{m} = \frac{0.5227}{\sqrt{N}} \qquad (2)$$

The speckle for 1 look amplitude SAR image obeys a Rayleigh distibution as in equation 3. An N look intensity speckle image is obtained by averaging N intensity single look images and is modelled as a Gamma distribution (equation 3.a). The multilook amplitude speckle can be obtained by averaging the N amplitude single look images or by averaging the N intensity images and then taking the square root (Frery et al., 1997). The latter follows a Square Root of Gamma distribution (Lee et al.,1994) as describes the equation 3.b. The former is described by the convolution of N Rayleigh distributions and for N=2 there is a closed form for it. Since there is no closed form for the distribution for N≥3 it is costumary to make an approximation (Yanasse et al., 1995) and describe it by the Square Root of Gamma distribution.

$$f(g) = \frac{g}{\sigma^2} e^{-\frac{g^2}{2\sigma^2}} \qquad (3)$$

where g is the random variable with parameter σ.

$$f(g) = \frac{\sigma}{\Gamma(\lambda)} (\sigma g)^{\lambda-1} e^{-\sigma g}, g > 0 \qquad (3.a)$$

where Γ(λ) is a value of the gamma function and g is the random variable with parameters σ and λ. For λ=1 the Gamma distribution is identical to the Exponential distribution. For $\lambda = n/2$ ($n>0$) and σ=1/2 the Gamma distribution is equivalent to a Chi-square distribution.

$$f(g) = \frac{2N^N}{\sigma^{2N}\Gamma(N)} (g)^{2N-1} e^{-\frac{Ng^2}{\sigma^2}}, g, N > 0 \qquad (3.b)$$

III. MAP Estimator

The MAP estimator of x is obtained by maximizing the "a posteriori" probability density function $f(x|z)$, which can be related to the "a priori" distribution $f(x)$ through equation (4). To simplify the notation, the indexes (A) in the following equations are dropped out. The conditional distribution *f(z|x)* which describes the model follows a Square Root of Gamma distribution is given by the equation (5).

$$f(x|z) = \frac{f(z|x)\,f(x)}{f(z)} \tag{4}$$

$$\frac{\partial \ln f(z\mid x)}{\partial x} + \frac{\partial \ln f(x)}{\partial x}\Big|_{x=\hat{x}_{MAP}} = 0 \tag{4.b}$$

$$f(z\mid x) = \frac{2N^N}{\sigma^{2N}\Gamma(N)}(x)^{2N-1} e^{-\frac{Nx^2}{\sigma^2}} \tag{5}$$

where N is the number of looks. This follows from the multiplicative model (equation 1) since given the signal *x,* the conditional probability density of *z* is a Square Root of Gamma density with mean value *x* (the mean value of *n* is one).

$$\frac{\partial \ln}{\partial x}[f(z|x)] = -\frac{2N}{x} + \frac{2z^2\Gamma^2(N+1/2)}{\Gamma^2(N)x^3} \tag{6}$$

We formulated several MAP filters using different “a priori” densities. These MAP equations are presented in the following.

III.1 Given the Gaussian “a priori” density

$$f(x) = \frac{1}{\sigma_x\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu_x}{\sigma_x}\right)^2} \tag{7}$$

The Gaussian MAP filter is given by the solution of the equation (8). This equation was obtained using the "a priori" knowledge from (7) combined with (6) in equation (5).

$$x^4\Gamma^2(N) - x^3\Gamma^2(N)\mu_x + x^2\Gamma^2(N)2N\sigma_x^2 - \sigma_x^2 2z^2\Gamma^2(N+1/2) = 0 \tag{8}$$

The estimators for μ_x and σ_x^2 are obtained by the following expressions:

$$\hat{\mu}_x = \hat{\mu}_z = \mu = \frac{1}{w}\sum_{i=1}^{w} z_i$$

$$\hat{\sigma}_x^2 = \frac{\sigma_z^2 - \hat{\mu}_z^2 . \sigma_n^2}{1+\sigma_n^2} \tag{9}$$

$$R = \frac{\hat{\sigma}_x^2}{\sigma_z^2}$$

where *w* is the size of window around the filtered pixel and σ_n^2 is the noise variance, which is a constant determined by the number of looks and the type of detection. *R* (Changle Li’s parameter) is the local ratio of original and noisy image variance. The set of expressions in equation (9) arose from the multiplicative model with unitary mean for the speckle noise.

Before filtering the noisy image, we calculate the local *R* (Li's ratio*)* parameter for all pixels using 3x3 windows. The one dimensional k-means algorithm over Li's ratio classifies pixels with similar statistics. Pixels assigned to the same cluster are filtered with the same window size for parameter estimation (3x3 or 5x5). The real and positive roots of the MAP equations whose values are between the mean and the observed pixel are taken as the filtered pixel values.

III. 2 Given the Gamma "a priori" density

$$f(x) = \frac{s_x}{\Gamma(l)}(s_x x)^{l-1} e^{-s_x x} \tag{10}$$

The MAP estimator is given by the solution of the equation

$$x^3\Gamma^2(N)s_x + x^2\Gamma^2(N)(2N+1-l) - 2z^2\Gamma^2(N+1/2) = 0 \tag{11}$$

$$\hat{l} = \frac{\hat{m}_x^2}{\hat{s}_x^2} \tag{12}$$

where the parameters s_x and l are estimated by the sample mean and the sample variance through the method of moments, using the multiplicative model. The estimated parameter $\hat{s}_x^2$ is the variance of the original signal calculated from the noisy signal through equation 9.

III.3 Given the Chi-square "a priori" density

$$f(x) = \frac{1}{2^{n/2}\Gamma(n/2)} x^{n/2-1} e^{-x/2} \tag{13}$$

where *n* denotes a Chi-square distribution with *n* degrees of freedom.

The MAP equation is given by

$$x^3\Gamma^2(N) + x^2\Gamma^2(N)[2+4N-n] - 4z^2\Gamma^2(N+1/2) = 0$$
$$n = m_x = m_z = \frac{1}{w}\sum_{k=1}^{w} z_k \tag{14}$$

III.4 Given the Exponential "a priori" density

$$f(x) = s_x e^{-s_x x} \tag{15}$$

The noisy pixel is updated with the solution of the MAP equation

$$x^3\Gamma^2(N)s_x + x^2\Gamma^2(N)2N - 2z^2\Gamma^2(N+1/2) = 0$$
$$\hat{s}_x = \frac{1}{\hat{m}_x} \qquad (16)$$

III.5 Given the Rayleigh “a priori” density

$$f(x) = \frac{x}{s_x^2} e^{\frac{-x^2}{2s_x^2}} \qquad (17)$$

The MAP estimator is given by the solution of the equation

$$x^4\Gamma^2(N) + x^2\Gamma^2(N)s_x^2(2N-1) - 2z^2 s_x^2\Gamma^2(N+1/2) = 0$$
$$\hat{s}_x = \hat{m}_x\sqrt{\frac{2}{p}} \qquad (18)$$

IV. Experimental Results

The original image in **Figure 1.a** is a piece of 481x481 pixels image of the National Forest of Tapajós, Pará, Brazil, taken on June, 26, 1993 by the JERS-1 satellite. It is a three looks, amplitude detected image. FIG. 1a

The presented filters were applied and their performance was evaluated in terms of the speckle reduction index, b **,** which is the ratio between the standard deviation and the mean over homogeneous areas. In **Table 1**, the estimated b indexes in a 41x41 pixels piece of the original image with initial coordinates (51,376) over a forest region are shown. The last row are the theoretical and practical values of b indexes over this region. The closeness of the theoretical and practical b coefficients implies that this forest area can be considered homogeneous. In the first column are the estimated b indexes in the MAP filtered area without k-means and in the second column are the b indexes in the MAP filtered area with the k-means algorithm. TAB. 1

MAP FILTER	$\beta=\sigma_z/\mu_z$ (without k-means)	$\beta=\sigma_z/\mu_z$ (with k-means)
GAUSSIAN	0.113	0.017
GAMMA	0.137	0.019
CHI-SQUARE	0.126	0.071
EXPONENTIAL	0.200	0.182
RAYLEIGH	0.192	0.162
HOMOGENEOUS REGION (41X41 PIXELS)	3 LOOKS THEORETICAL 0.2941	PRACTICAL 0.3029

Table 1-Estimated β indexes

V. Conclusions

The nonlinear, adaptive algorithms based on the MAP criterion proposed in this paper, besides decreasing substantially the standard deviation to the mean ratio improved the discrimination of the predominant classes (regeneration and forest) as shown by the histograms. The smoothing of the speckle in the Gaussian, Gamma, Chi-square, Exponential and Rayleigh MAP filtered images has been evaluated by the b index in **Table 1** and from the histograms. The indexes were calculated over an homogeneous area of forest (41x41 pixels). Some speckle reduction can be discerned in **Figures 2.a**, **3.a**, **4.a** and **5.a** which presented the best b indexes and in **Figures 3.b** and **5.b** the discrimination of classes has been improved by the use of the k-means algorithm. In the Chi-square MAP filtered image histogram, **Figure 9.b**, the classes are better discriminated than for the other distributions. The b indexes for the Exponential and Rayleigh MAP filters were the lowest, and even when using k-means the classes discrimination is not as evident as in **Figures 6.b** and **11.b**. Based on these results, the improvement obtained through the use of the k-means algorithm become clear.

FIG. 2a
FIG. 3a
FIG. 4a
FIG. 5a
FIG. 3b
FIG. 5b
FIG. 9b
FIG. 6b
FIG. 11b

VI. Further Developments

Future developments will use region growing techniques to determine windows with adaptive size and shape (not necessarily square) to estimate the local parameters of the MAP filters.

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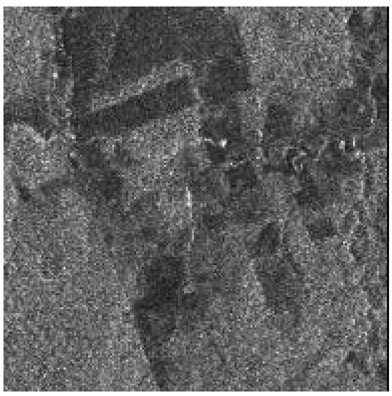
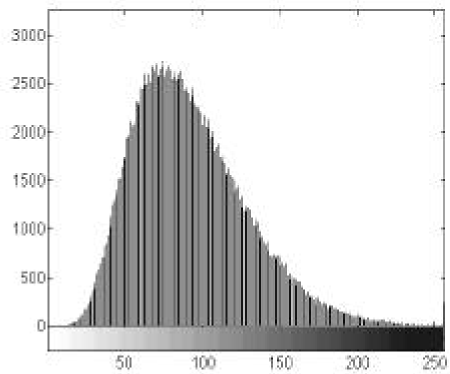
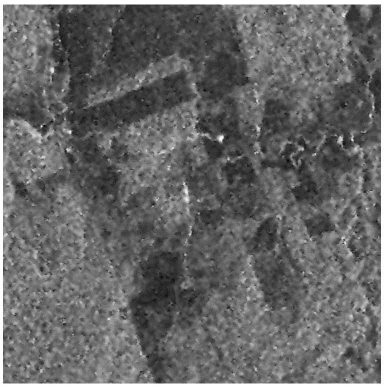


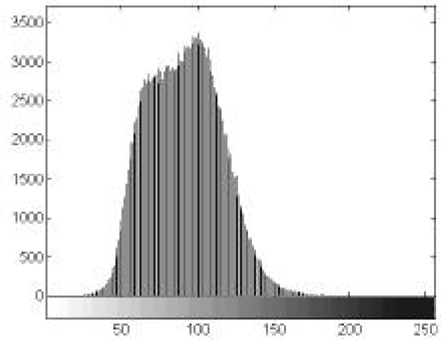
(1.a) ORIGINAL JERS-1 IMAGE



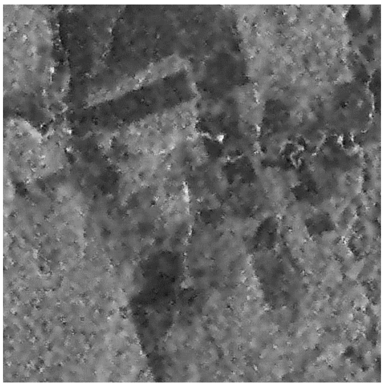
(1.b) ORIGINAL HISTOGRAM



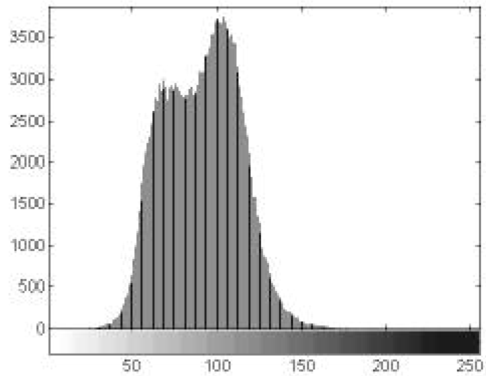
(2.a) GAUSSIAN MAP FILTERED
(without k-means)



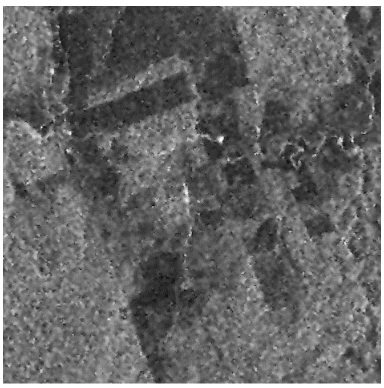
(2.b) GAUSSIAN MAP FILTERED HISTOGRAM
(without k-means)



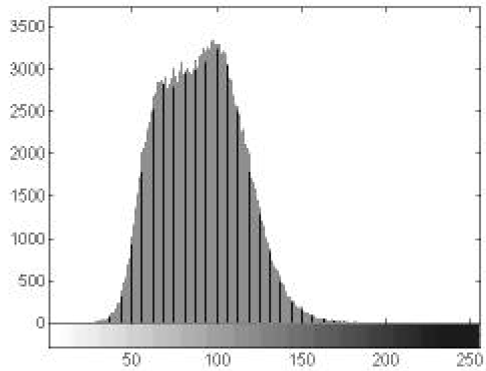
(3.a) GAUSSIAN MAP FILTERED IMAGE
(with k-means)



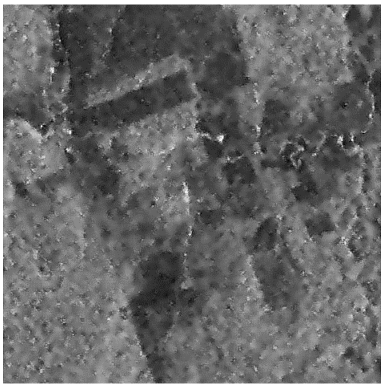
(3.b) GAUSSIAN MAP FILTERED IMAGE HISTOGRAM
(with k-means)



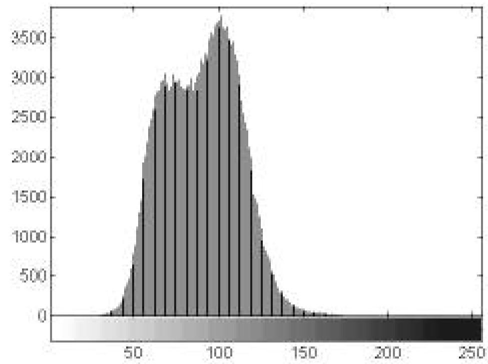
(4.a) GAMMA MAP FILTERED IMAGE
(without k-means)



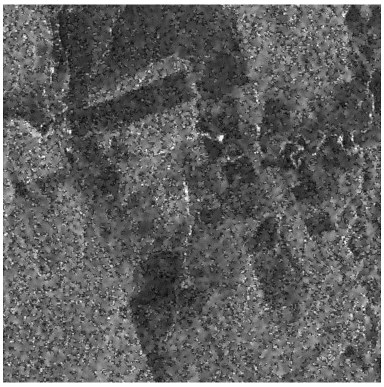
(4.b) GAMMA MAP FILTERED IMAGE HISTOGRAM
(without k-means)



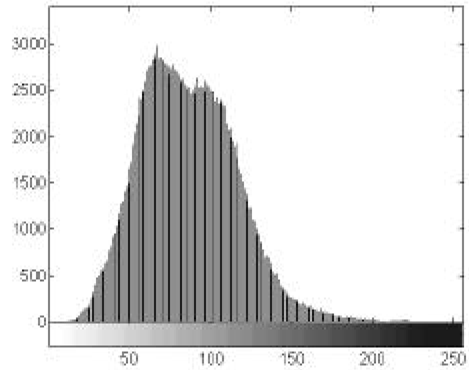
(5.a) GAMMA MAP FILTERED IMAGE (with k-means)



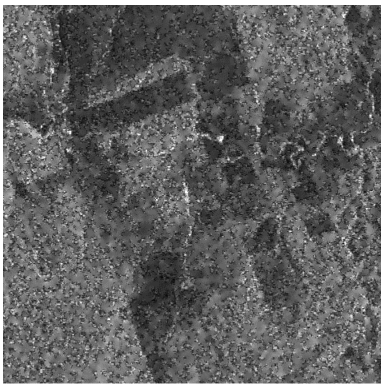
(5.b) GAMMA MAP FILTERED IMAGE HISTOGRAM (with k-means)



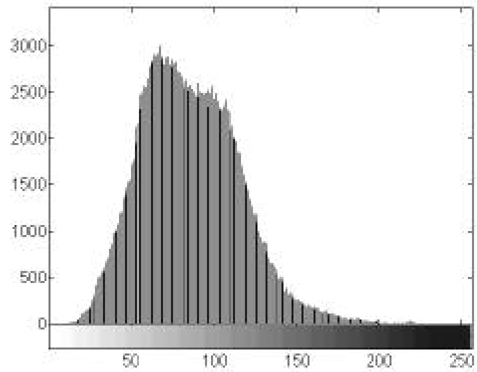
(6.a) EXPONENTIAL MAP FILTERED IMAGE (without k-means)



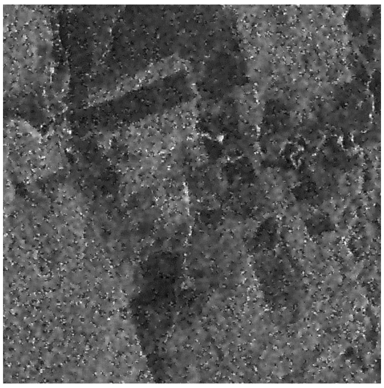
(6.b) EXPONENTIAL MAP FILTERED IMAGE HISTOGRAM (without k-means)



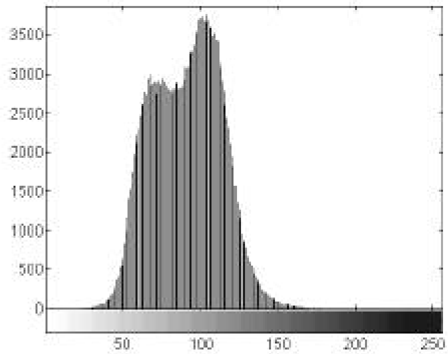
(7.a) EXPONENTIAL MAP FILTERED IMAGE (with k-means)



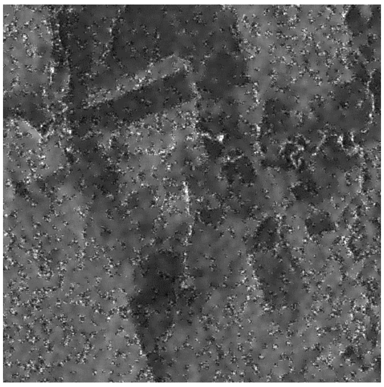
(7.b) EXPONENTIAL MAP FILTERED IMAGE HISTOGRAM (with k-means)



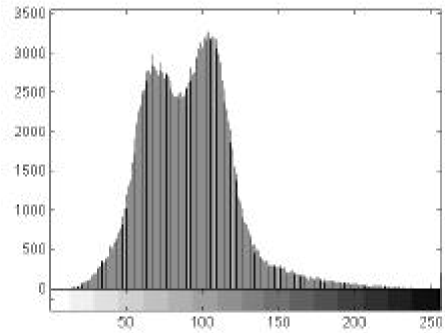
(8.a) CHI-SQUARE MAP FILTERED IMAGE (without k-means)



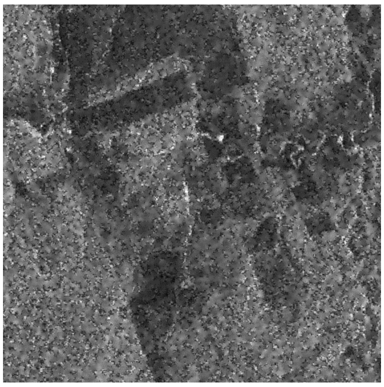
(8.b) CHI-SQUARE MAP FILTERED IMAGE HISTOGRAM (without k-means)



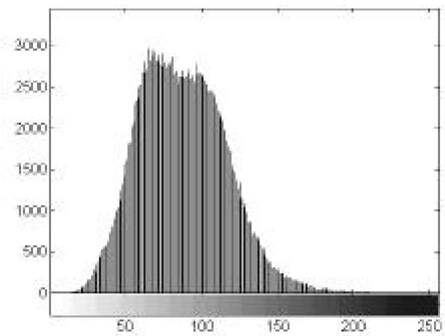
(9.a) CHI-SQUARE MAP FILTERED IMAGE (with k-means)



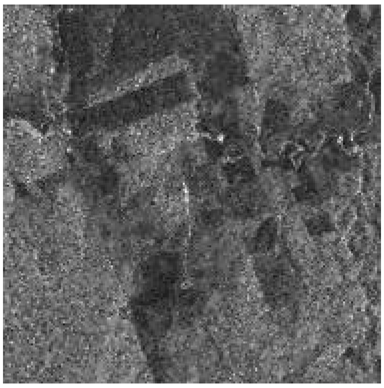
(9.b) CHI-SQUARE MAP FILTERED IMAGE HISTOGRAM (with k-means)



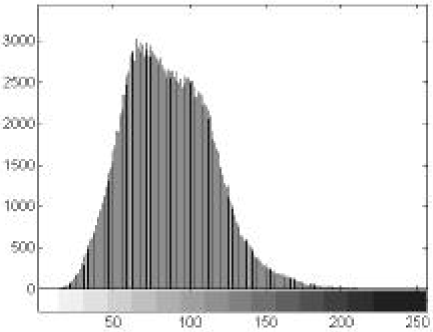
(10.a) RAYLEIGH MAP FILTERED IMAGE (without k-means)



(10.b) RAYLEIGH MAP FILTERED IMAGE HISTOGRAM (without k-means)



(11.a) RAYLEIGH MAP FILTERED IMAGE (with k-means)



(11.b) RAYLEIGH MAP FILTERED IMAGE HISTOGRAM (with k-means)